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One more source of information on the lepton mixing angles

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ABSTRACT

In the framework of the Standard Model with lepton mixing the radiative decay $\nu_i \to \nu_j \gamma$ of a neutrino of high ($E_\nu \sim 100\, GeV$) and super-high ($E_\nu \geq 1\, TeV$) energy is investigated in the Coulomb field of a nucleus. Estimates of the decay probability and "decay cross-section" for neutrinos of these energies in the electric field of nucleus permit one to discuss the general possibility of carrying out the neutrino experiment, subject to the condition of availability in the future of a beam of neutrinos of that high energies. Such an experiment could give unique information on mixing angles in the lepton sector of the Standard Model which would be independent of the specific neutrino masses if only the threshold factor $(1 - m_j^4/m_i^4)$ was not close to zero.

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At the present time the physics of the massive neutrino is the subject of intensive experimental and theoretical investigations. Given the neutrino's non-degenerate mass spectrum, it is natural to expect lepton mixing, similar to the well-known quark mixing, in the lepton sector of the electroweak theory. Neutrino oscillations [1] are the main source of information on the mixing angles rigidly correlate with the neutrino's mass spectrum (see, for example, [2]). In this work we come up with yet another source of information on the lepton mixing angles – namely, the radiative decay of a high-energy neutrino in an external electromagnetic field which has virtually no correlation with the mass spectrum. Such an intensive field may be represented, for example, by Coulomb field of a nucleus.

In our recent paper [3] we studied the neutrino radiative decay $\nu_i \to \nu_j \gamma$ in an external magnetic field within the Standard Model with lepton mixing. The effect of significant enhancement of the probability of the neutrino radiative decay by a magnetic field (magnetic catalysis) was discovered. It is important that the probability of the ultrarelativistic neutrino radiative decay in external field was found to be practically independent of the mass of the decaying neutrino, if only the decay channel is open ($m_i^2 \gg m_j^2$). The result we obtained for the amplitude of the ultrarelativistic neutrino radiative decay in an uniform magnetic field can be applied to handle the high-energy neutrino radiative decay in the Coulomb field of nucleus. Indeed, in this case the Coulomb field in the decaying neutrino rest frame appears, as does the magnetic field, very close to the crossed field ($\vec{\mathcal{E}} \perp \vec{B}, \mathcal{E} = B$). Here the non-uniformity of the electric field $\vec{\mathcal{E}}$ of the nucleus is of no consequence, because the loop process involved is "local" with the characteristic loop dimension $\Delta x \leq (E_\nu e \mathcal{E})^{-1/3}$ being significantly less than the nucleus size at neutrino energy $E_\nu \geq 100\, GeV$. In this case the decay amplitude we have obtained earlier (see Eq.(8) in Ref.[3]) can be brought into a more convenient form:

$$
\begin{aligned}
\mathcal{M} &\simeq \frac{e^2 G_F}{\pi^2} \, (\varepsilon^* \tilde{F} p) \left[(1-x) + \frac{m_j^2}{m_i^2}(1+x) \right]^{1/2} \sum_\ell K_{i\ell} K_{j\ell}^* \, J(\chi_\ell), \\
J(\chi_\ell) &= i \int_0^1 dt \; z_\ell \int_0^\infty du \; \exp\left[-i\,(z_\ell u + u^3/3) \right], \qquad (1) \\
z_\ell &= 4 \left[(1+x)(1-t^2) \left(1 - \frac{m_j^2}{m_i^2} \right) \chi_\ell \right]^{-2/3},
\end{aligned}
$$

where $F_{\mu\nu}$, $\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$ are the external electromagnetic field (Coulomb in our case) tensor and its dual tensor, p_μ, m_i are the 4-momentum and the mass of the initial neutrino, m_j is the mass of the final neutrino, ε_μ is the polarization 4-vector of the photon, $\chi_\ell = \sqrt{e^2 (pFFp)}/m_\ell^3$ is the so called dynamic parameter, m_ℓ is the mass of the virtual charged lepton, $K_{i\ell}$ is the lepton mixing unitary matrix ($\ell = e, \mu, \tau$), $x = \cos\theta$, θ is the angle between the vector $\vec{p}$ (momentum of the decaying

ultrarelativistic neutrino) and $\vec{q}_0$ (momentum of the photon in the decaying neutrino rest frame). When analyzing the amplitude (1), one should keep in mind that in the ultrarelativistic limit the kinematics of the decay $\nu_i(p) \to \nu_j(p') + \gamma(q)$ involves nearly parallel 4-momenta p, p' and q. Thus, the 4-vector of the neutrino current $j_\alpha = \bar{\nu}_j(p')\gamma_\alpha(1+\gamma_5)\nu_i(p)$ is also proportional to those vectors ($j_\alpha \sim p_\alpha \sim q_\alpha \sim {p'}_\alpha$). It is worth noting that the amplitude (1) is a sum of three loop contributions ($\ell = e, \mu, \tau$), each one being characterized by its "field form-factor" $J(\chi_\ell)$. The dependence of $J(\chi_\ell) = |J|\, e^{i\Phi}$ of the dynamical parameter χ_ℓ is represented in Fig. 1.

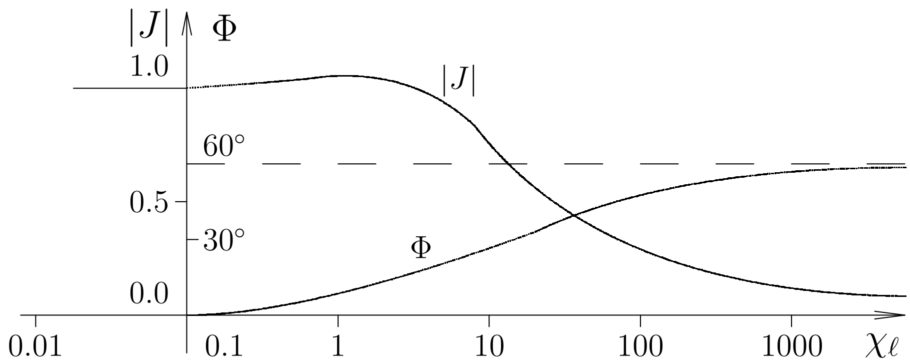

Fig. 1.

It is interesting to note that in a weak external field, when all $\chi_\ell \ll 1$, all field form-factors $J(\chi_\ell)$ are close to unit. In this case we obtain GIM weak field cancellation

$$\sum_\ell K_{i\ell}K^*_{j\ell}\, J(\chi_\ell) = \sum_\ell K_{i\ell}K^*_{j\ell} = 0, \qquad \text{at} \qquad i \neq j.$$

In an electric field this parameter can be represented as follows:

$$\chi_\ell \simeq \left(\frac{E_\nu}{m_\ell}\right)\left(\frac{e\mathcal{E}}{m_\ell^2}\right)\sin\alpha, \tag{2}$$

where α is the angle between the vector of the momentum $\vec{p}$ of the decaying neutrino and the electric field strength $\vec{\mathcal{E}}$. In a general way, cumbersome numerical calculations are required to find the probability. In the limit of super-high neutrino energies ($E_\nu \geq 1\, TeV$), however, the situation is drastically simplified, as at such neutrino energies the conditions $\chi_e \gg \chi_\mu \gg 1$, $\chi_\tau \ll 1$ are fulfilled in the vicinity of the nucleus. Therefore, it is sufficient for us to know only the asymptotics of the function $J(\chi)$:

$$\begin{aligned} J(\chi) &= 1 + O(\chi^2), \qquad \chi \ll 1, \\ J(\chi) &= O(\chi^{-2/3}), \qquad \chi \gg 1, \end{aligned} \tag{3}$$

so that the amplitude (1) is dominated by the contribution due to the virtual τ-lepton:

$$K_{ie}K^*_{je}\, J(\chi_e) + K_{i\mu}K^*_{j\mu}\, J(\chi_\mu) + K_{i\tau}K^*_{j\tau}\, J(\chi_\tau) \simeq K_{i\tau}K^*_{j\tau}.$$

Let us write in Table 1 the numerical values of the dynamical parameter χ_ℓ which correspond the electric field strength $\mathcal{E} \sim 10^{16}\, G$ in the vicinity of a nucleus $\mathcal{E} = Ze/r_N^2$.

Table 1.

$F_e = m_e^2/e \simeq 4 \cdot 10^{13}\, G$	$\chi_e = \dfrac{E_\nu}{m_e}\,\dfrac{\mathcal{E}}{F_e} \simeq 10^8$
$F_\mu = m_\mu^2/e \simeq 2 \cdot 10^{18}\, G$	$\chi_\mu = \dfrac{E_\nu}{m_\mu}\,\dfrac{\mathcal{E}}{F_\mu} \simeq 10^2$
$F_\tau = m_\tau^2/e \simeq 10^{20}\, G$	$\chi_\tau = \dfrac{E_\nu}{m_\tau}\,\dfrac{\mathcal{E}}{F_\tau} \simeq 0.1$

This permits employing the expression for the decay probability we have obtained earlier (see Eq.(13) in Ref.[3]) which can be written in the following form:

$$w \simeq \frac{\alpha}{4\pi}\, \frac{G_F^2}{\pi^3}\, E_\nu e^2 \mathcal{E}^2 \sin^2\alpha\, \left(1 - \frac{m_j^4}{m_i^4}\right)\, |K_{i\tau}K^*_{j\tau}|^2. \tag{4}$$

It is intriguing to compare this expression for the probability with the well-known expression for the probability of the radiative decay $\nu_i \to \nu_j\gamma$ of a high-energy neutrino in vacuum (see, for example, [4]):

$$w_0 \simeq \frac{27\alpha}{32\pi}\, \frac{G_F^2}{192\pi^3}\, \frac{m_i^6}{E_\nu}\, \left(\frac{m_\tau}{m_W}\right)^4\, \left(1 - \frac{m_j^4}{m_i^4}\right)\, \left(1 - \frac{m_j^2}{m_i^2}\right)\, |K_{i\tau}K^*_{j\tau}|^2. \tag{5}$$

The comparison of the formula demonstrates the enhancing influence of the external field on the radiative decay

$$R \equiv \frac{w}{w_0} \simeq \frac{512}{9}\, \left(\frac{m_W}{m_\tau}\right)^4\, \left(\frac{E_\nu}{m_i}\right)^2\, \left(\frac{e\mathcal{E}}{m_i^2}\right)^2\, \sin^2\alpha. \tag{6}$$

As an illustration, let us give a numerical estimate of R in the case of the decay of a neutrino of energy $E_\nu \sim 1\, TeV$ in the vicinity of a nucleus with the atomic number $Z \sim 20$:

$$R \simeq 2 \cdot 10^{+61}\, \left(\frac{1\, eV}{m_i}\right)^6\, \left(\frac{E_\nu}{1\, TeV}\right)^2. \tag{7}$$

The neutrino radiative decay in substance must result in γ-quantum of energy $E_\gamma \sim E_\nu$ being observed as the decay product. In experiment, this process would

appear as inelastic scattering of the neutrino on the nucleus. Using the expression (4) for the probability and taking the nucleus as a uniformly charged sphere of radius r_N we obtain the following expression for the "cross-section" of the decay of a super-high-energy neutrino ($E_\nu \geq 1\,TeV$) in the electric field of a nucleus with the atomic number Z:

$$\begin{aligned} \sigma &\simeq \frac{4}{5} Z^2 \left(\frac{\alpha}{\pi}\right)^3 \frac{G_F^2 E_\nu}{r_N} \left(1 - \frac{m_j^4}{m_i^4}\right) |K_{i\tau} K_{j\tau}^*|^2 \\ &\simeq 10^{-44}\, Z^2 \left(\frac{10^{-12} cm}{r_N}\right) \left(\frac{E_\nu}{1\,TeV}\right) |K_{i\tau} K_{j\tau}^*|^2 \quad (cm^2). \end{aligned} \tag{8}$$

A cumbersome numerical calculation of the probability at more realistic energies (e.g., at $E_\nu \sim 100\,GeV$, for which $J(\chi_\mu)$ is not small) shows that Eq.(8) in this case also gives a correct estimate of the "cross-section" within an order of magnitude. This requires the following substitution in Eq.(8):

$$|K_{i\tau} K_{j\tau}^*|^2 \longrightarrow \sin^2 \Theta_{12}\ \cos^2 \Theta_{12}\ \overline{|J(\chi_\mu)|^2}, \tag{9}$$

where Θ_{12} is the mixing angle of ν_1 and ν_2 (analogous to the Kabibbo angle in the quark sector) and $\overline{|J(\chi_\mu)|^2} \sim 1$ is the average of the modulus squared of the "field form-factor" of the muon loop.

It is worthwhile noting that the "cross-section" (8)-(9) we have presented is, of course, numerically small, but not so small as not to allow discussion concerning the possibility of such an experiment in the future. The "cross-section" measurement at this level of accuracy would supply unique information on mixing angles in the lepton sector of the electroweak theory independent of the specific neutrino masses, if only the threshold factor $(1 - m_j^4/m_i^4)$ was not close to zero.

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